New Parabolized Navier-Stokes Code for Three-Dimensional Chemically Reacting Flows

D. K. Prabhu* and J. C. Tannehill†
Iowa State University, Ames, Iowa
and
J. G. Marvin‡
NASA Ames Research Center, Moffett Field, California

Abstract

A NEW parabolized Navier-Stokes (PNS) code has been developed to compute the laminar hypersonic chemically reacting flow around three-dimensional bodies. This new space-marching code solves the gasdynamic and species conservation equations in a fully coupled manner using a noniterative, implicit, approximately factored, finite-difference algorithm. The space-marching method is made well-posed by special treatment of the streamwise pressure gradient term. The code has been used to compute the hypersonic laminar flow over cones at various angles of attack. The flow medium in these calculations is a multicomponent mixture of O_2, N_2, O, N, NO, NO^+, and e^-. The results of these calculations have been compared with the results from reacting boundary-layer calculations and show excellent agreement.

Contents

The numerical computation of hypersonic, viscous, chemically reacting, three-dimensional flowfields is of current research interest. Of the many methods employed, space-marching methods require much less computer time than time-marching methods and provide accurate solutions in applicable cases. The PNS equations in the space-marching methods category have been used widely in computing ideal-gas flowfields. Bhutta et al.[1] were one of the first to compute chemical nonequilibrium flows using these equations. They solved the chemistry and gasdynamics separately and used an iterative approach to couple the two. In contrast to their approach, Prabhu et al.[2] developed a two-dimensional/axisymmetric chemical nonequilibrium PNS code in which the gas dynamic and chemical conservation equations were solved simultaneously in a noniterative manner. The success of this code has led to the development of a new PNS code for three-dimensional bodies. The development of this new PNS code is described briefly here; more details can be found in Ref. 3.

The PNS equations are obtained from the complete Navier-Stokes equations by 1) neglecting the time derivatives, 2) neglecting the viscous and diffusion terms in the streamwise direction, and 3) retaining only a fraction of the streamwise pressure gradient in the subsonic region of the flow.[2,3] These equations for a three-dimensional laminar flow of an n-component gas, expressed in (ξ,η,ζ) computational coordinates via the transformation $\xi=\xi(x,y,z)$, $\eta=\eta(x,y,z)$, and $\zeta=\zeta(x,y,z)$, can be written in nondimensional, strong conservation-law form as

$$\frac{\partial \boldsymbol{E}^i}{\partial \xi}+\frac{\partial \boldsymbol{F}^i}{\partial \eta}+\frac{\partial \boldsymbol{G}^i}{\partial \zeta}=\frac{1}{Re}\frac{\partial \boldsymbol{F}^v}{\partial \eta}+\boldsymbol{W}^c$$

where $\boldsymbol{E}^i$", $\boldsymbol{F}^i$, and $\boldsymbol{G}^i$ are the $(n+4)$-component transformed inviscid fluxes. The transformed viscous flux vector is $\boldsymbol{F}^v$, and this $(n+4)$-component vector contains derivatives with respect to the η coordinate only due to the thin-layer approximation. The vector of chemical production/depletion terms is represented by $\boldsymbol{W}^c$. These vectors and source terms are functions of the elements (and their spatial derivatives) of $\boldsymbol{Q}$, and this $(n+4)$-component vector of conservation (dependent) variables is chosen as[3]

$$\boldsymbol{Q}=\{\rho,\rho u,\rho v,\rho w,\rho H,\rho c_1,\rho c_2,\ldots,\rho c_{n-1}\}^T$$

The chemical model used in the present calculations is air consisting of O_2, O, N, NO, NO^+, N_2, and e^-. The electrons are eliminated from the species set by using the law of conservation of charge. All of the pertinent data have been obtained from Ref. 4.

The algorithm used to solve the PNS equations is an adaptation of the one developed by Tannehill et al.[5] This approximately factored algorithm is implicit, noniterative, first-order accurate in the ξ direction, and second-order accurate in the η and ζ directions. The implicit part of the algorithm leads to a block-tridiagonal system of equations in the η and ζ directions. The blocks are square matrices of order $(n+4)$. For the seven-species air model considered in the present calculations, the blocks are square matrices of order 10. Any discontinuities in the flowfield are "captured" as a part of the solution. At the outer boundary, freestream conditions are imposed. No-slip and zero normal pressure gradient boundary conditions are applied at the wall. Furthermore, the wall is assumed to be either isothermal or adiabatic, and either catalytic or noncatalytic. At the pitch plane of symmetry, reflection boundary conditions are imposed; thus, flow symmetry is maintained for flows without yaw. The PNS equations require initial conditions in addition to the boundary conditions. The usual procedure is to use an initial data surface generated by a full Navier-Stokes code. For conical or pointed bodies, however, the code generates its own starting solution. This starting solution is generated iteratively using a "stepback" procedure.[2,3]

In order to validate the present nonequilibrium PNS code, the hypersonic laminar flow of dissociating air was computed over a 10-deg cone at 0-, 2.5-, 5-, 7.5-, and 10-deg angles of attack. The altitude chosen for these calculations was 60.96 km, where the ambient pressure and temperature are 20.35 N/m^2 and 252.6 K, respectively. The remaining flow conditions are as follows: $V_\infty=8100$ m/s, $T_w=1200$ K and noncatalytic wall, $c_{O2}=0.2629$, $c_{N2}=0.7371$, and $\mathcal{L}_e=1.4$.

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*Research Assistant, Department of Aerospace Engineering and Computational Fluid Dynamics Center; currently, Research Scientist, Eloret Institute, Sunnyvale, CA. Member AIAA.

†Manager, Computational Fluid Dynamics Center, and Professor, Department of Aerospace Engineering. Associate Fellow AIAA.

‡Chief, Experimental Fluid Dynamics Branch. Associate Fellow AIAA.

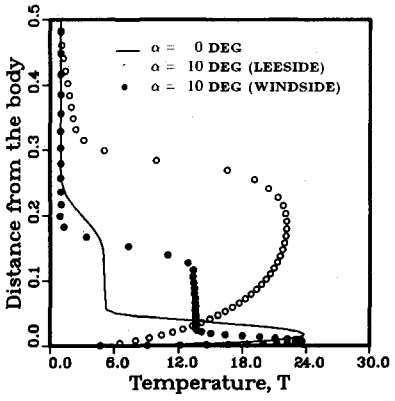

Fig. 1 Temperature profiles at $x/L = 2.5$, $\alpha = 10$ deg.

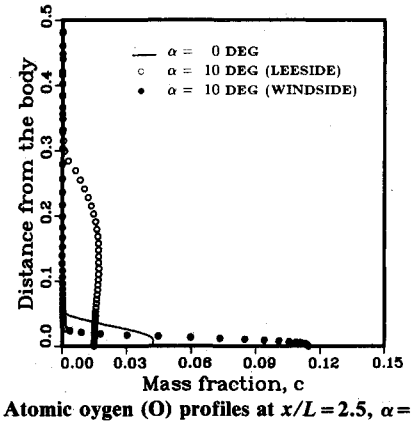

Fig. 2 Atomic oygen (O) profiles at $x/L = 2.5$, $\alpha = 10$ deg.

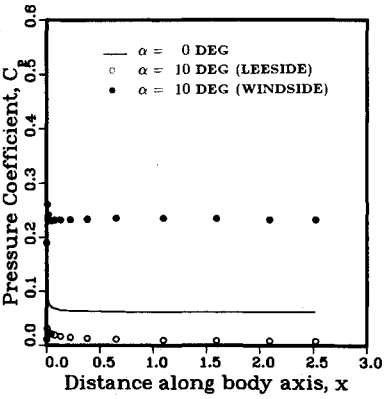

Fig. 3 Axial variation of wall pressure, $\alpha = 10$ deg.

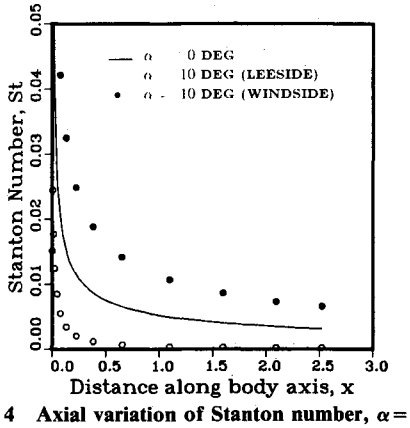

Fig. 4 Axial variation of Stanton number, $\alpha = 10$ deg.

The computation for each angle of attack was started at $x = 1.5 \times 10^{-3}$ using the starting solution generated by the "stepback" procedure. The marching step size was increased proportionally to the maximum thickness of the subsonic layer. The grid used in the calculations consisted of 67 points in the normal direction and 21 points in the meridional direction. The distance of the first point away from the body surface was varied linearly along the streamwise direction. The grid lines were placed normal to the body, and the height of the outer boundary was kept at 0.5. The mesh spacing on the windside had to be refined progressively for each angle of attack to properly resolve the boundary layer on the windside of the body. The starting solution was marched up to an axial location of $x/L = 3.5$ for the 0-deg angle-of-attack case and up to $x/L = 2.5$ for the other cases. The results of the 0-deg angle-of-attack calculation were compared to those obtained from the reacting boundary-layer code of Ref. 4 and found to be in excellent agreement. These results can be found in Ref. 3.

The computed results for the angle-of-attack calculations are too numerous to show in their entirety. Only sample results of the 10-deg angle-of-attack calculations are shown. The results of the 2.5-, 5-, and 7.5-deg angle-of-attack computations can be found in Ref. 3. Since there is a paucity of experimental data in such severe hypersonic regimes, the results of the angle-of-attack calculations are simply compared with the results of the 0-deg angle-of-attack calculations. In order to provide greater details of the flowfield, only 50% of the solution domain is shown in the figures. The effect of angle of attack on the temperature is depicted in Fig. 1. The temperature profiles in the pitch plane of symmetry are shown in these figures; the following observations can be made: 1) the boundary layer thickens considerably on the leeside while thinning on the windside, 2) the edge temperatures increase rapidly on the windside while decreasing gradually on the leeside, and 3) the peak temperatures decrease on the leeside but stay very nearly the same on the windside. The shock on the leeside weakens and begins to disappear.

In Fig. 2, the effect of angle of attack on a typical chemistry variable such as the mass fraction atomic oxygen is shown. On the lee plane of symmetry, the amount of atomic oxygen at the wall decreases with increasing angle of attack. However, as a result of diffusional effects, atomic oxygen is present over a large distance from the body. On the windside, the amount of atomic oxygen increases at the wall but is present at a smaller distance. These results are not contrary to what one would expect since the leeside is generally much cooler than the windside. The axial variation of the surface pressure and Stanton number on the windward and leeward meridians is shown in Figs. 3 and 4, respectively: The increase in surface pressure and heat transfer on the windside and the decrease on the leeside with increasing angle of attack are evident.

These calculations were performed not only to demonstrate the three-dimensional capabilities of the code but also to provide "benchmark" calculations for future code developers. The computations were performed on the CRAY-2 (NAS) computer at NASA Ames Research Center and required 0.43 ms per grid point per step. The 10-deg angle-of-attack calculation required about 600 steps. The present study indicates that there is a need for a comprehensive experimental database for validation of nonequilibrium codes.

Acknowledgment

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